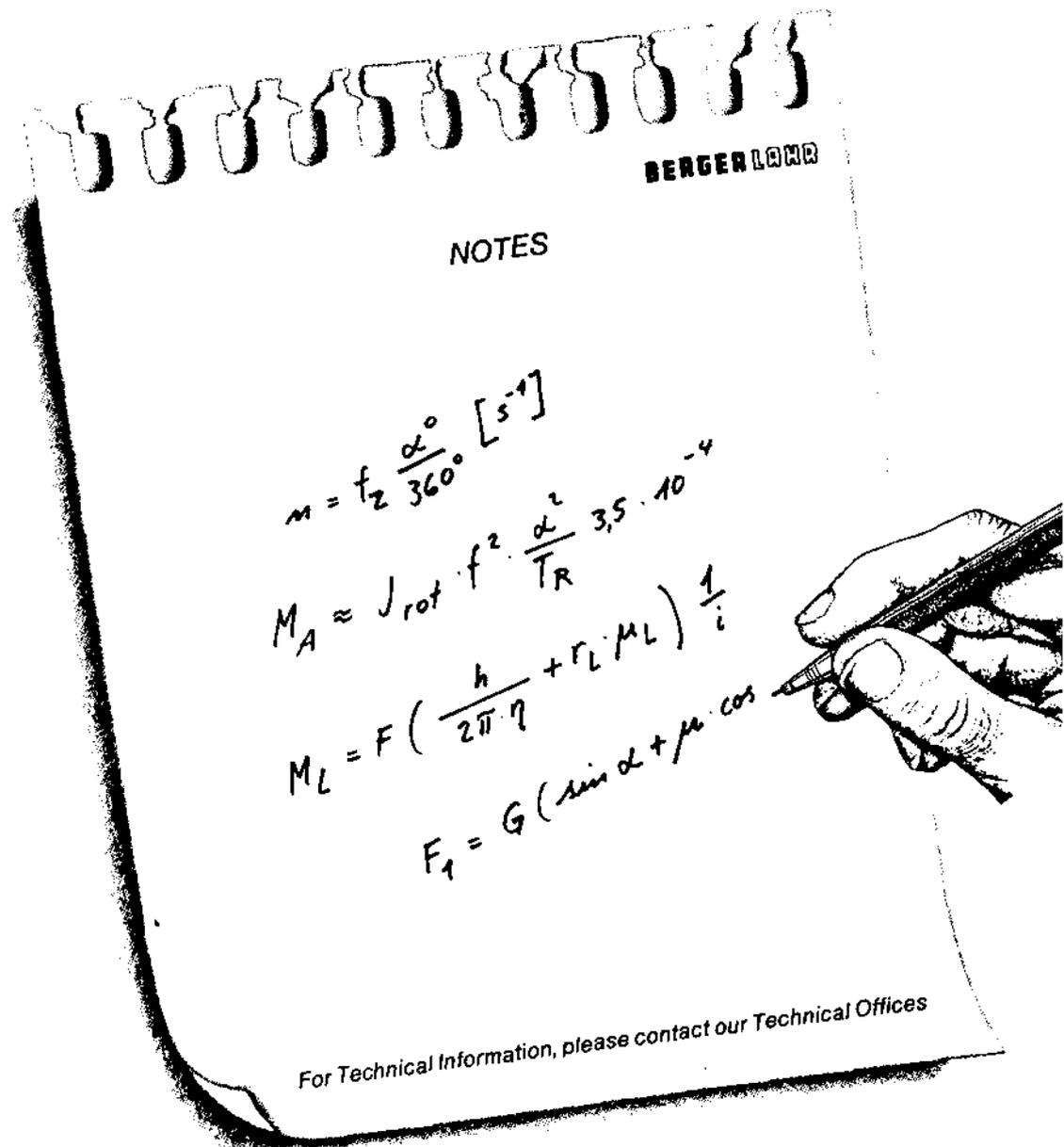


# Formulas + Calculations for Optimum Selection of a Stepmotor

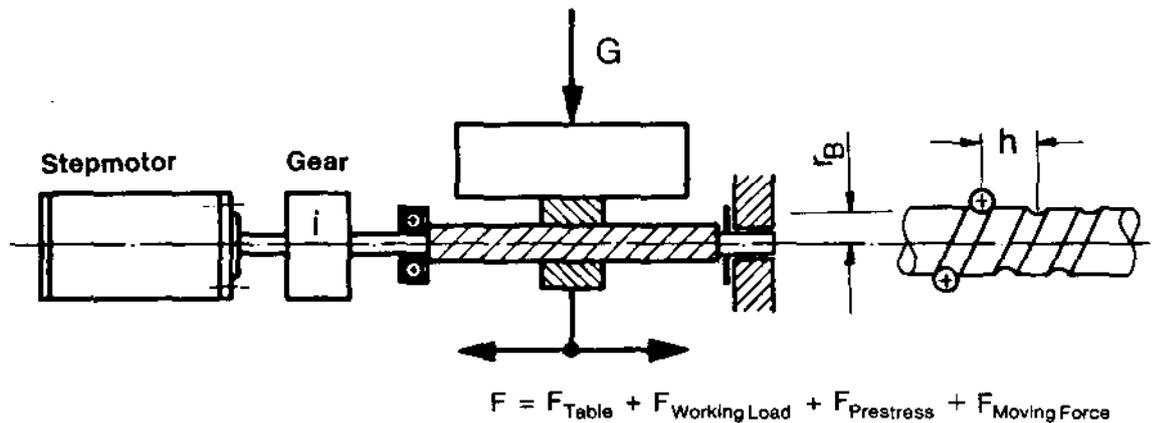


# 1. General Formulas

*Vismère*

## 1.1 Spindle Drive

### 1.1.1 Determination of Load Torque $M_{Load}$



The required drive torque of a spindle drive is determined by the sum of the load torques and the required acceleration torque.

(1)  $M_{Motor} = M_{Load} + M_{Accel}$  [Ncm]

(2)  $M_{Load} = F \left( \frac{h}{2\pi \cdot \eta} + r_B \cdot \mu_B \right) \frac{1}{i}$  [Ncm]

$F$  = Total force on nut [N]

$h$  = Spindle pitch [cm]

$r_B$  = Spindle bearing mean radius [cm]

$\mu_B$  = Spindle bearing friction coefficient

$i$  = Gear ratio =  $n_{Motor} / n_{Spindle}$

$\eta = \frac{\tan \alpha}{\tan (\alpha + \rho)}$  = Efficiency coefficient of converting M into an axial force

### 1.1.2 Experience Values

$\eta = 0.9$  for ball bearing spindles (see figure)

$\eta = 0.3$  for steel spindles with bronze nut

$r_B \cdot \mu_B = 0.015$  cm for roller bearing

$r_B \cdot \mu_B = 0.15$  cm for steel/bronze friction bearings

**$F_{Prestress}$**

At 10% Prestress and  $h = 5$  mm: approx. 11 to 15 N

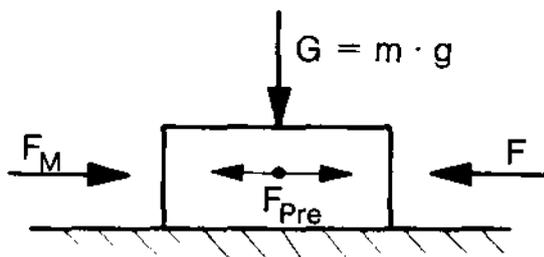
At 20% Prestress and  $h = 5$  mm: approx. 22 to 30 N

At 10% Prestress and  $h = 10$  mm: approx. 40 to 60 N

At 20% Prestress and  $h = 10$  mm: approx. 80 to 120 N

### 1.1.3 Determination of Total Load F on Spindle Nut

#### a) Vertically Acting Forces

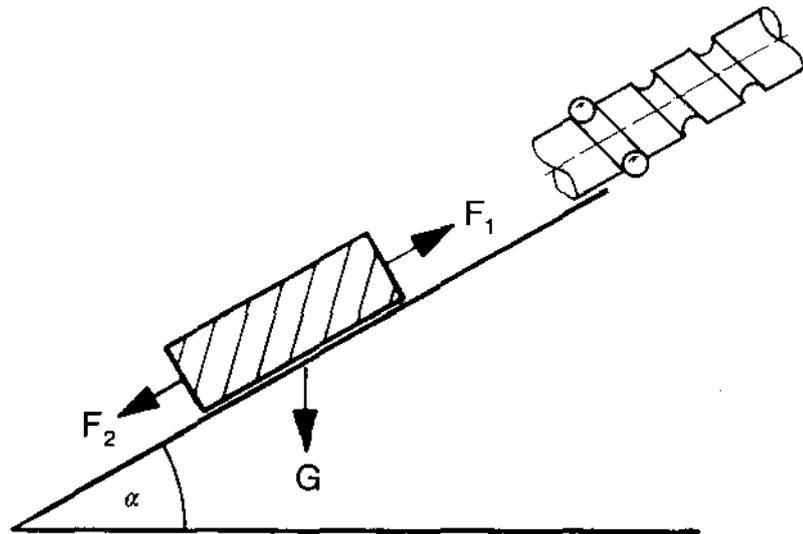


(3) 
$$F = \mu \cdot G + F_M + F_{Pre} \quad [N]$$

- G = Weight of carriage and structure [kg]
- $F_M$  = Moving Force [N]
- $F_{Pre}$  = Prestress when using springloaded counter nut
- $\mu$  = Friction coefficient
- G  $\approx$  10 · m [N]
- m = Mass [kg]

Values for $\mu$	Dry	Lubricated
Steel on steel	0,18	0,12
Steel on cast iron	0,19	0,10
Steel on bronze	0,11	0,10
Axial Guide		
Rolling friction, Rollers	-	0,005

b) „Non-vertical” Spindle



(4)  $F = F_{Pre} + F_M + F_{1,2}$  [N]

(4a)  $F_1 = G (\sin \alpha + \mu \cdot \cos \alpha)$  if moving up [N]

(4b)  $F_2 = G (\sin \alpha - \mu \cdot \cos \alpha)$  if moving down [N]

For upward motion

(4c)  $F = F_{Pre} + F_M + G (\sin \alpha + \mu \cdot \cos \alpha)$  [N]

### 1.1.4 Determination of Moments of Inertia

The total moment of inertia  $J_{tot}$  is the sum of the moments of inertia of all masses in rotatory and translatory motion.

$$(5) \quad J_{ext} = J_{rot} + J_{trans} \quad [\text{kgcm}^2]$$

$J_{ext}$  = Total external J referenced to motor shaft

$J_{rot}$  = Rotatory moment of inertia (full cylinder)

$J_{trans.}$  = Translatory moment of inertia

$$(5a) \quad J_{tot} = J_{ext} + J_{Mot.} \quad [\text{kgcm}^2]$$

#### a) Rotatory Moment of Inertia $J_{rot}$ – Full Cylinder

$$(6) \quad J_{rot} = \frac{1}{2} \pi \cdot r^4 \cdot L \cdot \gamma \quad [\text{kgcm}^2]$$

$r$  = Radius of spindle cm

$L$  = Length cm

$\gamma$  = Specific weight  $\text{kg/cm}^3$

Steel  $\gamma = 7,85 \cdot 10^{-3} \text{ kg/cm}^3$

Aluminum  $\gamma = 2,7 \cdot 10^{-3} \text{ kg/cm}^3$

Brass  $\gamma = 8,4 \cdot 10^{-3} \text{ kg/cm}^3$

For Steel

$$(7) \quad J_{rot} = 7,72 \cdot 10^{-4} \cdot d^4 \cdot L \quad [\text{kgcm}^2]$$

For Aluminum

$$(7a) \quad J_{rot} = 2,7 \cdot 10^{-4} \cdot d^4 \cdot L \quad [\text{kgcm}^2]$$

#### b) Translatory Moment of Inertia $J_{trans}$

$$(8) \quad J_{trans} = m \left( \frac{h}{2\pi} \right)^2 \quad [\text{kgcm}^2]$$

$m$  = Moved mass in kg

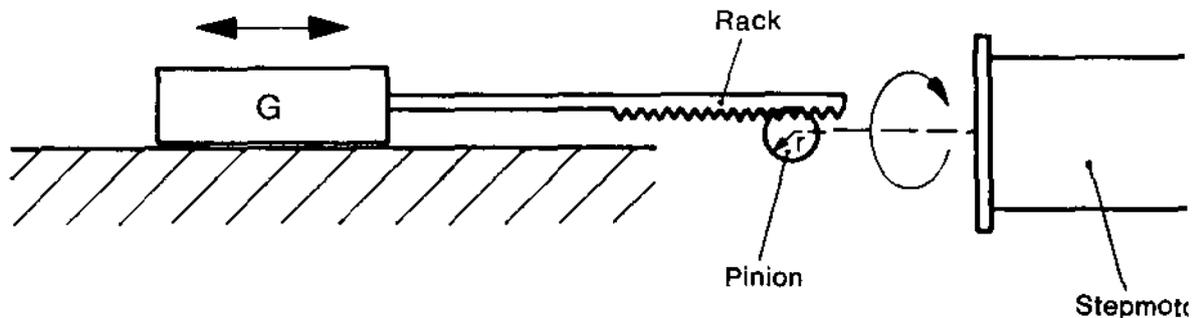
$h$  = Spindle pitch in cm

If a reduction gear is used, the external moment of inertia  $J_{ext}$  is reduced by the square of the gear ratio.

$$(9) \quad J_{ext} = (J_{rot} + J_{trans}) \frac{1}{i^2} \quad [\text{kgcm}^2]$$

## 1.2 Rack and Pinion Drive

Horizontally moved mass driven by rack and pinion.



### 1.2.1 Total Torques

The motor must provide the following total torques:

- Acceleration of weight G, including rack
- Acceleration of pinion
- Acceleration of rotor
- Overcoming the friction

#### 1.2.1.1 Moments of Inertia

The following formula is used to calculate the rotatory moment of inertia equivalent to the weight

$$(19) \quad J_{eq} = G \cdot r^2 \quad [\text{kgcm}^2]$$

G = Weight in kg

r = Radius in cm

$$(6) \quad J_{rot} = \frac{1}{2} \cdot \pi \cdot r^4 \cdot L \cdot \gamma \quad [\text{kgcm}^2] \quad J_{rot} = J_{Pin}$$

$$(20) \quad J_{tot} = J_{eq} + J_{Pin} + J_{rot} \quad [\text{kgcm}^2]$$

#### 1.2.1.2 Acceleration and Load Torques

$$(18) \quad M_A = J_{tot} \cdot \frac{f \cdot 2 \cdot \pi \cdot \alpha}{t_A \cdot 360^\circ \cdot 10^2} \quad [\text{Ncm}]$$

$$(21) \quad M_{tot} = M_A + M_L \quad [\text{Ncm}]$$

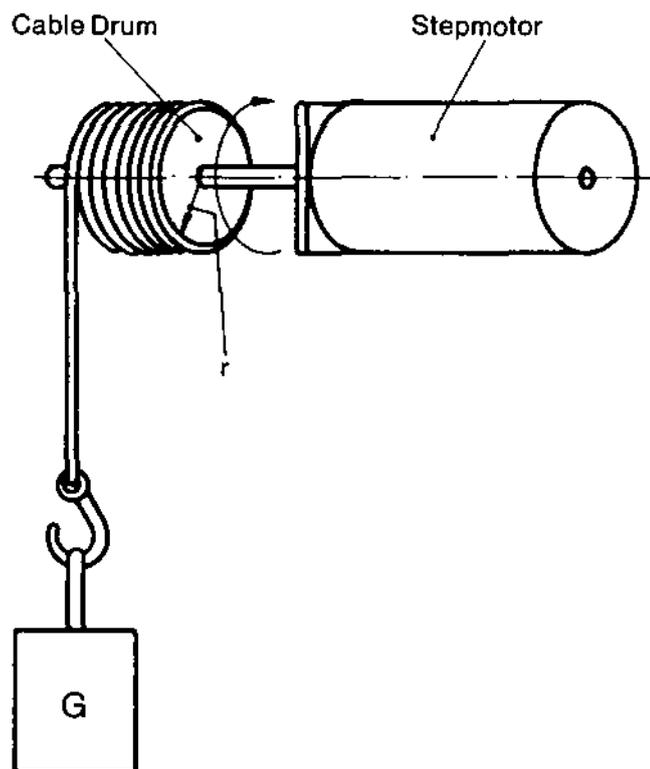
$$(22) \quad M_L \approx G \cdot \mu \cdot r \quad [\text{Ncm}]$$

### 1.3 Drum Drive

#### Lifting a Weight by Cable and Drum

##### 1.3.1 Total Torques

- a) Acceleration of the weight
- b) Acceleration of the drum
- c) Lifting of the Weight G



##### 1.3.1.1 Moments of Inertia

$J_{eq}$  = Equivalent moment of inertia

The same formulas as used for the rack and pinion drive apply.

$$(19) \quad J_{eq} = G \cdot r^2 \quad [\text{kgcm}^2]$$

$$(6) \quad J_{Drum} = \frac{1}{2} \cdot \pi \cdot r^4 \cdot L \cdot \gamma \quad [\text{kgcm}^2]$$

##### 1.3.1.2 Acceleration and Load Torques

$$(18) \quad M_A = J_{tot} \cdot \frac{f \cdot 2 \cdot \pi \cdot \alpha}{t_A \cdot 360^\circ \cdot 10^2} \quad [\text{Ncm}]$$

$M_A$  = Torque required for accelerating the system

$$(23) \quad M_L = G \cdot r \quad [\text{Ncm}]$$

$M_L$  = Torque required for lifting the weight

$$(21) \quad M_{tot} = M_A + M_L \quad [\text{Ncm}]$$

$M_{tot}$  = Total torque required for lifting the weight

## 1.4 Additional Formulas

### 1.4.1 Start-Stop Operation

$$(10) \quad M_A \approx J_{\text{tot}} \cdot f^2 \cdot \frac{\alpha^2}{T_R} \cdot 3,5 \cdot 10^{-4} \quad [\text{Ncm}]$$

$$(10a) \quad f_{\text{Start}} \approx 199 \cdot \sqrt{\frac{M_A}{J_{\text{tot}}}} \quad \left[ \frac{1}{\text{s}} \right]$$

$J_{\text{tot}}$  = Total Moment of Inertia [kgcm<sup>2</sup>]  
consisting of  $J_{\text{ext}} + J_{\text{Motor}}$

$f$  = Step frequency [Hz]

$\alpha$  = Step angle [Degrees]

$T_R$  = Division angle of rotor teeth  
(for 5-phase motors: 7.2°)

$M_A$  =  $M_{\text{Mot}} - M_L$  [Ncm]

$M_{\text{Mot}}$  = M at actual step frequency used

### 1.4.2 Calculation of Time for a Linear Acceleration ( $t_A$ )

$$(11) \quad t_A = J_{\text{tot}} \cdot \frac{2 \cdot \pi \cdot \alpha \cdot f}{360^\circ \cdot M_A \cdot 10^2} \quad [\text{s}]$$

$J_{\text{tot}}$  = Total moment of inertia consisting of  $J_{\text{ext}} + J_{\text{Mot}}$  [kgcm<sup>2</sup>]

$\alpha$  = Step angle [Degrees]

$f$  = Desired operating frequency [Hz]

$M_A$  = Acceleration torque [Ncm]

$$(11a) \quad M_A = M_{\text{Mot}} (\text{at } f) - M_L \quad [\text{Ncm}]$$

### 1.4.3 Torques

$$(2) \quad M_L = F \left( \frac{h}{2 \cdot \pi \cdot \eta} + r_B + \mu_B \right) \frac{1}{i} \quad [\text{Ncm}] \quad \text{Spindle drive}$$

$$(22) \quad M_L \sim G \cdot \mu \cdot r \quad [\text{Ncm}] \quad \text{Rack and pinion drive}$$

$$(23) \quad M_L = G \cdot r \quad [\text{Ncm}] \quad \text{Drum drive}$$

$M_L$  = Load torque

#### 1.4.4 Advance, Speed and Power

Distance Increment

$$(12) \quad \Delta s = \frac{h}{Z \cdot i} \quad [\text{cm}]$$

$h$  = Spindle pitch in cm

$Z$  = Number of steps per revolution

$i$  = Gear ratio

Advance Speed

$$(13) \quad v = \Delta s \cdot f = \frac{h}{Z \cdot i} f \quad [\text{cm/s}] \quad \left( \begin{array}{l} \text{for spindle drives} \\ v = \text{rate of feed} \end{array} \right)$$

$f$  = Step frequency [ $\text{s}^{-1}$ ]

$$(13a) \quad f = \frac{v \cdot Z \cdot i}{2 \cdot \pi \cdot r} \quad [\text{s}^{-1}] \quad \left( \begin{array}{l} \text{for rotatory drives} \\ v = \text{peripheral speed} \end{array} \right)$$

Advance in a defined time  $t$

$$(14) \quad s = \Delta s \cdot f \cdot t \quad [\text{cm}]$$

$t$  = time in s

Motor RPM

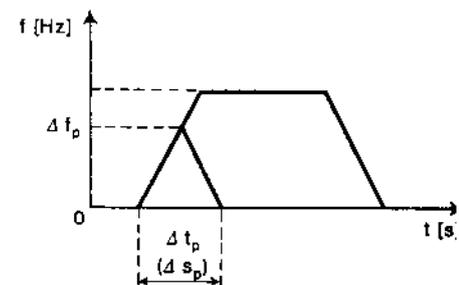
$$(15) \quad n = \frac{60 \cdot f}{Z \cdot i} \quad [\text{min}^{-1}]$$

Power for rotation

$$(16) \quad P = 0,00105 M \cdot n \quad [\text{W}]$$

$M$  in Ncm

$n$  in  $\text{min}^{-1}$



Positioning time for short distances

$$(24) \quad \Delta t_p = \frac{t_A \cdot \Delta s_p}{s_A} \quad [\text{s}]$$

Positioning frequency for short distances

$$(25) \quad \Delta f_p = \frac{f \cdot \Delta t_p}{t_A} \quad [\text{s}^{-1}]$$

$$(26) \quad s_{A \text{ in cm}} = s_{A \text{ in steps}} \cdot \Delta s \quad [\text{cm}]$$

$\Delta t_p$  = Positioning time [s]

$\Delta s_p$  = Positioning distance [cm]

$\Delta f_p$  = Max. positioning freq. [ $\text{s}^{-1}$ ]

$s_{A \text{ in cm}}$  = Acceleration distance [cm]

$s_A$  = Acceleration distance in ir

#### 1.4.5 Determination of the Moment of Inertia J for Arbitrary Bodies by Means of Measurement

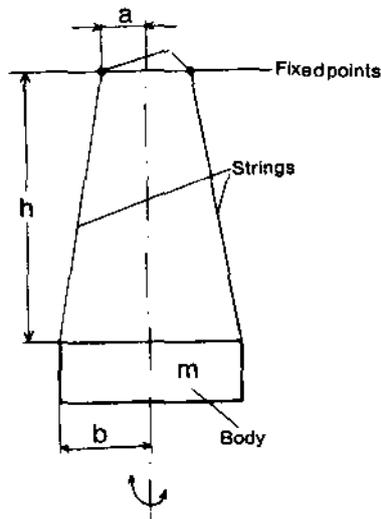
##### Procedure

The body is freely suspended by two strings attached to two fixed points. Then it is brought into rotational oscillation about the sketched center line. The moment of inertia is found using the previously determined mass  $m$  of the body and the distances  $a$ ,  $b$  and  $h$  in the following formula.

$$(17) \quad J = 25 \cdot T^2 \cdot m \cdot \frac{a \cdot b}{h} \quad [\text{gcm}^2]$$

$J$  = Moment of inertia in  $\text{gcm}^2$

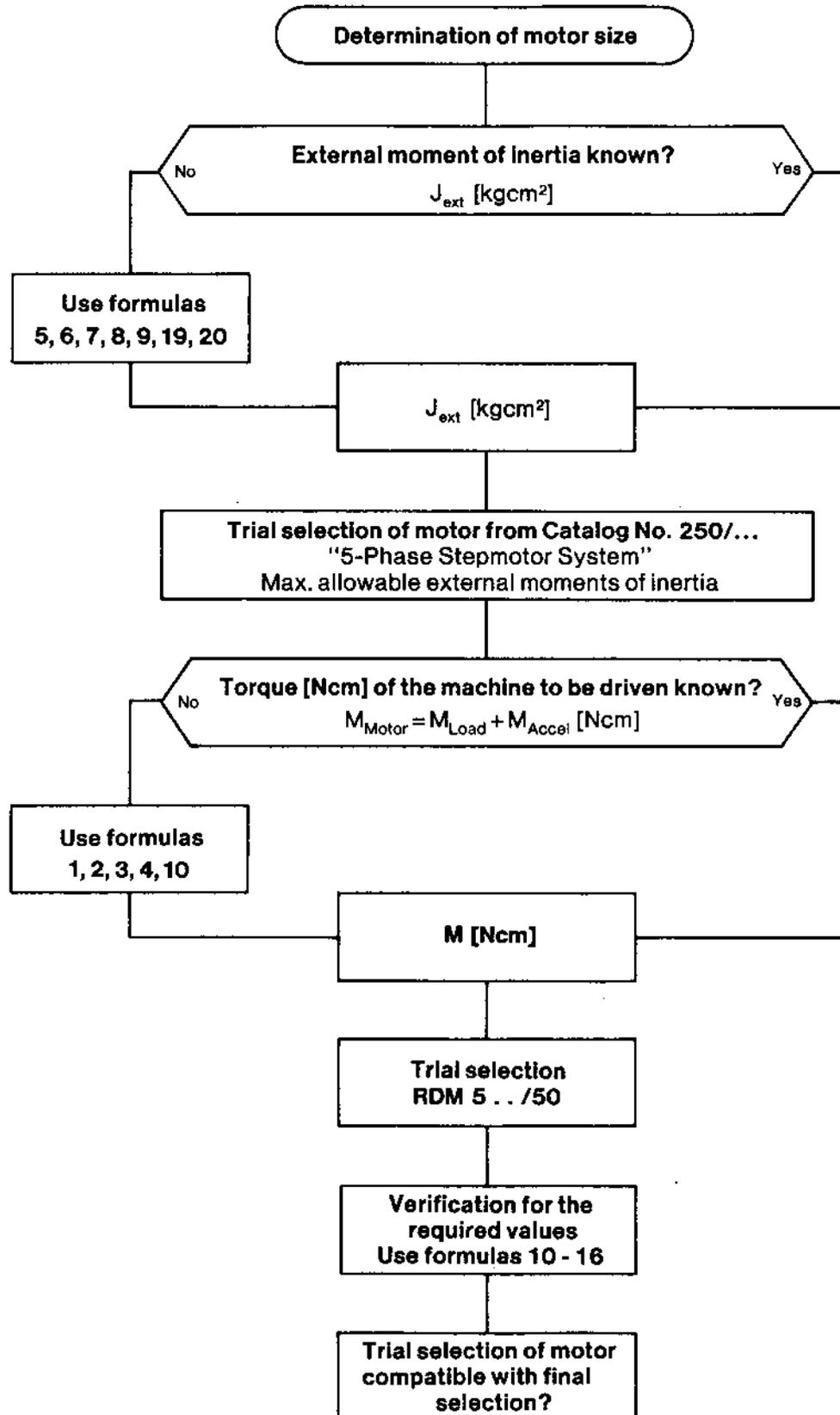
$T$  = Duration of period in sec.



$\left. \begin{array}{l} a \\ b \\ h \end{array} \right\} = \text{Distances in cm}$

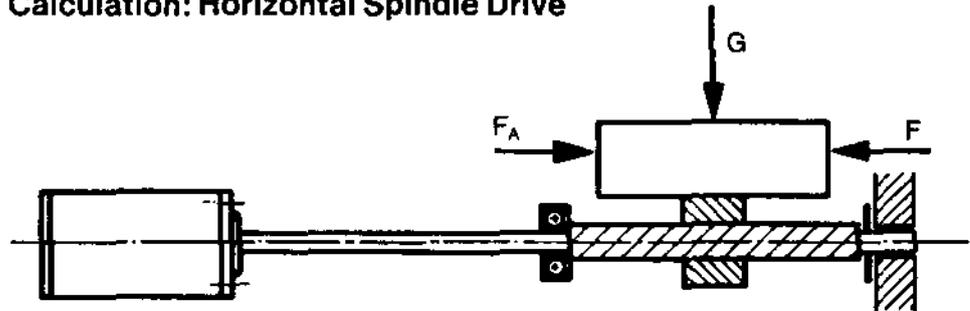
### 1.4.6 How to Find the Optimum Stepmotor

- Aids: 1. "Formulas and Calculations for Optimum Selection of a Stepmotor" (this brochure)  
 2. BERGER Catalog "5-Phase Stepmotor System"



## 2. Calculation Examples

### 2.1 Calculation: Horizontal Spindle Drive



#### 2.1.1 Known and Required Values

Known Values

$$G = 1000 \text{ N}$$

$$F_A = 250 \text{ N}$$

$$\mu = 0,1$$

$$\eta = 0,9$$

Required Values

$$v_{\text{slew}} = 12 \text{ m/min} = 20 \text{ cm/s}$$

$$\text{Positioning time for } 10 \text{ mm} = 0,5 \text{ s}$$

$$\text{Resolution: } 0,01 \text{ mm}$$

$$\text{Spindle diameter} = 35 \text{ mm}$$

$$\text{Spindle length} = 800 \text{ mm}$$

$$\text{Spindle pitch } h = 5 \text{ mm}$$

$$\text{Travel distance} = 700 \text{ mm}$$

Sought: the correct stepmotor

#### 2.1.2 Required Torque

$$(2) \quad M_L = F \left( \frac{h}{2 \pi \cdot \eta} + r_B \cdot \mu_B \right) \frac{1}{i} \quad [\text{Ncm}]$$

$$(3) \quad F = \mu \cdot G + F_A + F_{\text{Pre}} \quad [\text{N}]$$

$$F = 0,1 \cdot 1000 \text{ N} + 250 \text{ N} = 350 \text{ N}$$

$$M_L = 350 \text{ N} \left( \frac{0,5 \text{ cm}}{2 \cdot 3,14 \cdot 0,9} + 0,015 \text{ cm} \right) = 36 \text{ Ncm}$$

$$M_L = 36 \text{ Ncm}$$

The calculation example does not include gear ratio  $\left(\frac{1}{i}\right)$  and  $F_{\text{Pre}}$

### 2.1.3 Existing Moments of Inertia

$$(5) \quad J_{\text{ext}} = J_{\text{rot}} + J_{\text{trans}} \quad [\text{kgcm}^2]$$

$$(6) \quad J_{\text{rot}} = 0,5 \cdot \pi \cdot r^4 \cdot L \cdot \gamma \quad [\text{kgcm}^2]$$

$$J_{\text{rot}} = 0,5 \cdot 3,14 \cdot (1,75 \text{ cm})^4 \cdot 80 \text{ cm} \cdot \frac{7,85 \text{ kg}}{\text{cm}^3} \cdot 10^{-3}$$

$$J_{\text{rot}} = 9,25 \text{ kgcm}^2$$

$$(8) \quad J_{\text{trans}} = m \left( \frac{h}{2 \pi} \right)^2 \quad [\text{kgcm}^2]$$

$$J_{\text{trans}} = 100 \text{ kg} \left( \frac{0,5}{2 \cdot 3,14} \right)^2 \quad [\text{kgcm}^2]$$

$$J_{\text{trans}} = 0,63 \text{ kgcm}^2$$

$$(5) \quad J_{\text{ext}} = J_{\text{rot}} + J_{\text{trans}}$$

$$J_{\text{ext}} = 9,25 \text{ kgcm}^2 + 0,63 \text{ kgcm}^2 = 9,88 \text{ kgcm}^2 \approx 10 \text{ kgcm}^2$$

$$J_{\text{ext}} = 10 \text{ kgcm}^2$$

### 2.1.4 Required Operating Frequency

$$(13) \quad v = \frac{h}{Z \cdot i} \cdot f \quad [\text{cm/s}]$$

$$f = \frac{v \cdot Z \cdot i}{h} = \frac{20 \text{ cm/s} \cdot 500 \text{ Steps/Rev.}}{0,5 \text{ cm}} \quad [\text{s}^{-1}]$$

$$f = 20000 \text{ s}^{-1}$$

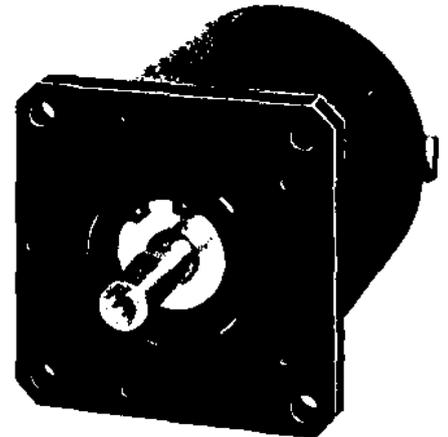
### 2.1.5 Values Determining the Motor Size

1.  $M_L = 36 \text{ Ncm}$
2.  $J_{\text{ext}} = 10 \text{ kgcm}^2$
3.  $f = 20000 \text{ Hz}$

## 2.1.6 Determination of Motor Size

### 5-Phase Stepmotor Data Overview

The motor data are contained in our Catalog No. 250/... »5-Phase Stepmotor System« in the following arrangement.



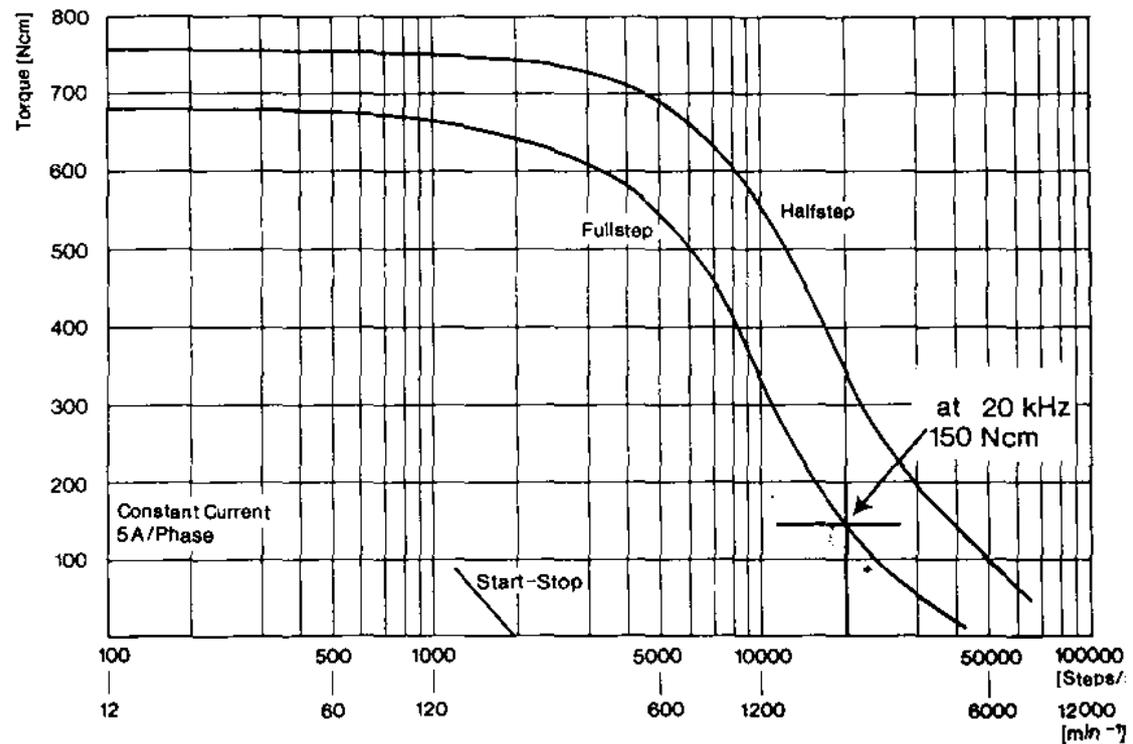
„Size 110“

MOTOR MODEL	RDM 51117/50	RDM 51122/50
Step angle [Degrees]	0,72	0,72
Maximum torque [Ncm]	700	1000
Holding Torque, excited [Ncm]	750	1100
Max. Power [W] at ..... Hz	430 15	465 5
Moment of inertia of rotor [kgcm <sup>2</sup> ]	7,5	11,5

### Motor Characteristics (Constant-Current Operation / Standard Winding)

Motor Model RDM 51117/50

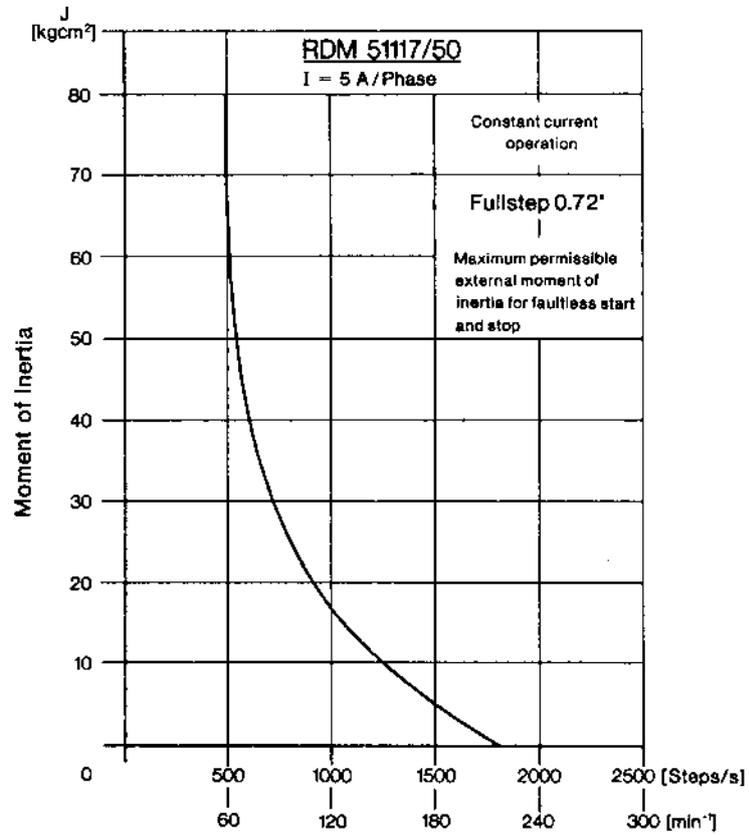
$R_w = 0,3 \Omega$   $I_w = 5 A$



\* For fullstep of 0.72°

## Maximum Permissible External Moments of Inertia

Motors of Series 511 .. /50



Motor size determined from curves: RDM 51117/50

Motor data see Catalog 250 "5-Phase Stepmotor System"

$$J_{\text{Motor}} = 7,5 \text{ kgcm}^2$$

$$M_A = M_{\text{Mot}} (\text{at } 20 \text{ kHz}) - M_L \text{ [Ncm]}$$

$$M_A = 150 \text{ Ncm} - 36 \text{ Ncm} = 114 \text{ Ncm}$$

### 2.1.7 Determination of Acceleration Time

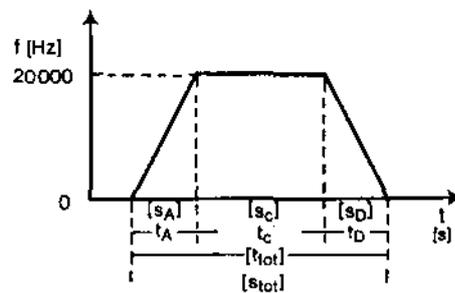
For linear acceleration and deceleration, the acceleration and deceleration times are equal.

$$(5a) \quad J_{\text{tot}} = J_{\text{Rot}} + J_{\text{trans}} + J_{\text{Mot}} = 17,5 \text{ kgcm}^2$$

$$(11) \quad t_A = J_{\text{tot}} \cdot \frac{2\pi \cdot \alpha \cdot f}{360^\circ \cdot M_A \cdot 10^2} \quad [\text{s}]$$

$$t_A = 17,5 \text{ kgcm}^2 \cdot \frac{2 \cdot 3,14 \cdot 0,72^\circ \cdot 20000 \text{ Hz}}{360^\circ \cdot 114 \text{ Ncm} \cdot 10^2}$$

$$t_A = 0,386 \text{ s} \sim 0,39 \text{ s}$$



$t_A$  = Time for acceleration  
 $t_C$  = Time for constant speed  
 $t_D$  = Time for deceleration  
 $t_{\text{tot}}$  = Total travel time

$s_A$  = Steps for acceleration  
 $s_C$  = Steps for constant speed  
 $s_D$  = Steps for deceleration

### 2.1.8 Distance Traveled in Total Travel Time

$$(27) \quad s_A = \frac{f \cdot t_A}{2} \quad [\text{Distance in steps}]$$

$$s_A = \frac{20000 \cdot 0,39 \text{ s}}{2} = 3900 \text{ Steps}$$

During acceleration phase = 3900 Steps =  $s_A$   
 During deceleration phase = 3900 Steps =  $s_D$

Sum for acceleration and deceleration = 7800 Steps

Total distance traveled  $s_{\text{tot}} = 700 \text{ mm} \triangleq 70\,000 \text{ Steps}$

### 2.1.9 Total Travel Time

$$t_{\text{tot}} = t_A + t_C + t_D \quad [\text{s}]$$

$$t_C = \frac{s_{\text{tot}} - (s_A + s_D)}{f} \quad [\text{s}]$$

$$t_C = \frac{70\,000 - (3900 + 3900)}{20\,000}$$

$$t_C = 3,11 \text{ s}$$

$$t_{\text{tot}} = t_A + t_C + t_D \quad [\text{s}]$$

$$t_{\text{tot}} = 0,39 + 3,11 + 0,39$$

$$t_{\text{tot}} = 3,89 \text{ s}$$

## 2.1.10 Verification of Required Values

### 2.1.10.1 Positioning Time $\Delta t_p$ [s]

$\Delta t_p$  required 0.5 s

$$(24) \quad \Delta t_p = \frac{t_{A[s]} \cdot \Delta s_p [cm]}{s_A [cm]} \quad [s]$$

$$\Delta s_p = 10 \text{ mm} = 1 \text{ cm}$$

$$(26) \quad s_A [cm] = s_A [\text{Steps}] \cdot \Delta s = s_A \cdot \frac{h}{Z}$$

$$s_A [cm] = 3900 \text{ Steps} \cdot \frac{0,5 \text{ cm}}{500 \text{ Steps}} = 3,9 \text{ cm}$$

$$\Delta t_p = \frac{0,39 \text{ s} \cdot 1 \text{ cm}}{3,9 \text{ cm}} = 0,1 \text{ s}$$

### 2.1.10.2 Smallest Distance Increment per Step

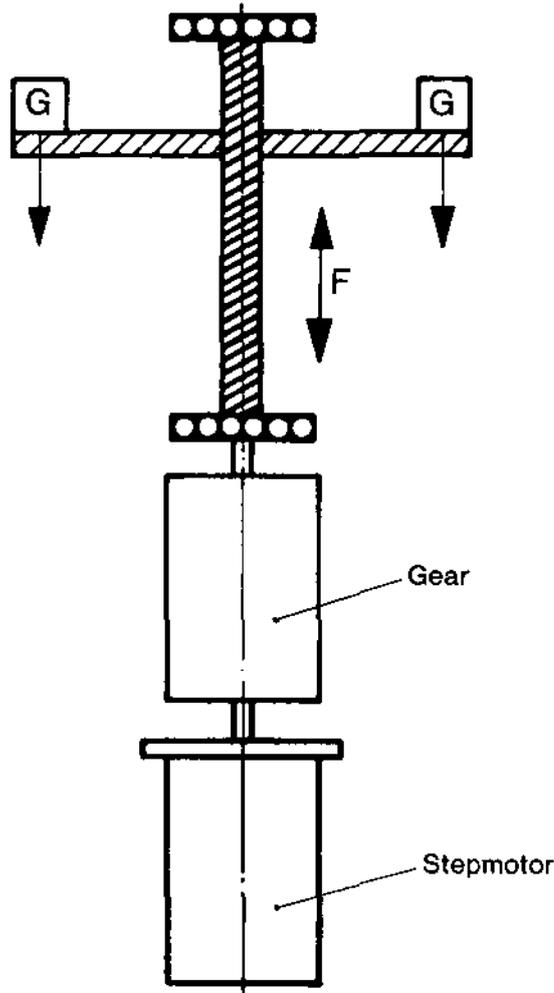
$\Delta s$  required 0.01 mm

$$(12) \quad \Delta s = \frac{h}{Z \cdot i} \quad [cm]$$

$$\Delta s = \frac{0,5}{500 \cdot 1} = 0,001 \text{ cm} = 0,01 \text{ mm}$$

## 2.2 Calculation: Vertical Spindle Drive (Lifting Force)

### 2.2.1 Known and Required Values



#### Known Values

$$G = 150 \text{ kg}$$

$$\eta = 0,9$$

$$\mu = 0,1$$

Spindle diameter	63 mm
length	10 m
pitch	10 mm
Gear ratio	$i = 20 : 1$

#### Required Values

Positioning time 10 mm in 1 s

Resolution < 0.01 mm

### 2.2.2 Required Torque

$$(2) \quad M_L = F \left( \frac{h}{2\pi \cdot \eta} + r_B \mu_B \right) \cdot \frac{1}{i} \quad [\text{Ncm}]$$

$$(4c) \quad F = G \cdot (\sin \alpha + \mu \cdot \cos \alpha) \quad [\text{N}]$$

$$\text{at } \alpha = 90^\circ$$

$$\sin \alpha = 1$$

$$\cos \alpha = 0$$

$$F = 1500 \text{ N} (1 + 0,1 \cdot 0)$$

$$F = 1500 \text{ N}$$

$$M_L = 1500 \text{ N} \left( \frac{1 \text{ cm}}{2 \pi \cdot \eta} + r_B \cdot \mu_B \right) \cdot \frac{1}{i} \text{ [Ncm]}$$

$$M_L = 1500 \text{ N} (0,177 + 0,015) \frac{1}{20} = 14,4 \text{ Ncm}$$

### 2.2.3 Existing Moments of Inertia

$$(9) \quad J_{\text{ext}} = \left( J_{\text{rot}} + J_{\text{trans}} \right) \cdot \frac{1}{i^2}$$

$$(6) \quad J_{\text{rot}} = 0,5 \cdot \pi \cdot r^4 \cdot L \cdot \gamma \text{ [kgcm}^2\text{]}$$

Moment of Inertia of Spindle

$$J_{\text{rot}} = 0,5 \cdot 3,14 (3,15 \text{ cm})^4 \cdot 1000 \text{ cm} \cdot 7,85 \text{ kg/cm}^2 \cdot 10^{-3}$$

$$J_{\text{rot}} = 1213,42 \text{ kgcm}^2$$

$$(8) \quad J_{\text{trans}} = m \left( \frac{h}{2 \pi} \right)^2 \text{ [kgcm}^2\text{]}$$

(equivalent moment of inertia of weight G)

$$J_{\text{trans}} = 150 \text{ kg} \left( \frac{1 \text{ cm}}{2 \cdot 3,14} \right)^2 = 3,8 \text{ kgcm}^2$$

$$J_{\text{ext}} = (J_{\text{rot}} + J_{\text{trans}}) \cdot \frac{1}{i^2} \text{ [kgcm}^2\text{]}$$

$$J_{\text{ext}} = \left( 1213,42 \text{ kgcm}^2 + 3,8 \text{ kgcm}^2 \right) \cdot \frac{1}{20^2}$$

$$J_{\text{ext}} = 3,043 \text{ kgcm}^2$$

### 2.2.4 Required Speed

In order to determine the exact motor data, the speed must be determined.

Required: 10 mm in 1 s  $\hat{=} 1 \text{ s}^{-1}$

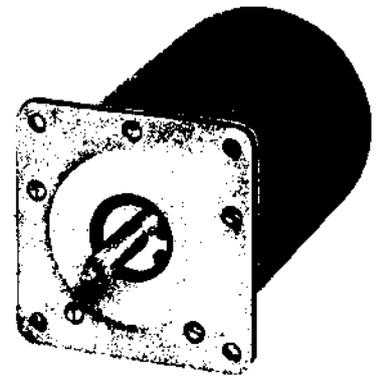
at  $i = 20 : 1 \hat{=} n_{\text{Mot}} \hat{=} 1200 \text{ min}^{-1} \hat{=} 10 \text{ kHz}$

Hence, the motor torques have to be determined for a step frequency of 10 kHz.

## 2.2.5 Values Determining the Motor Size

- 1)  $M_L = 14,4 \text{ Ncm}$
- 2)  $J_{\text{ext}} = 3,043 \text{ kgcm}^2$
- 3)  $f = 10000 \text{ Hz}$

## 2.2.6 Determination of Motor Size



"Size 90"

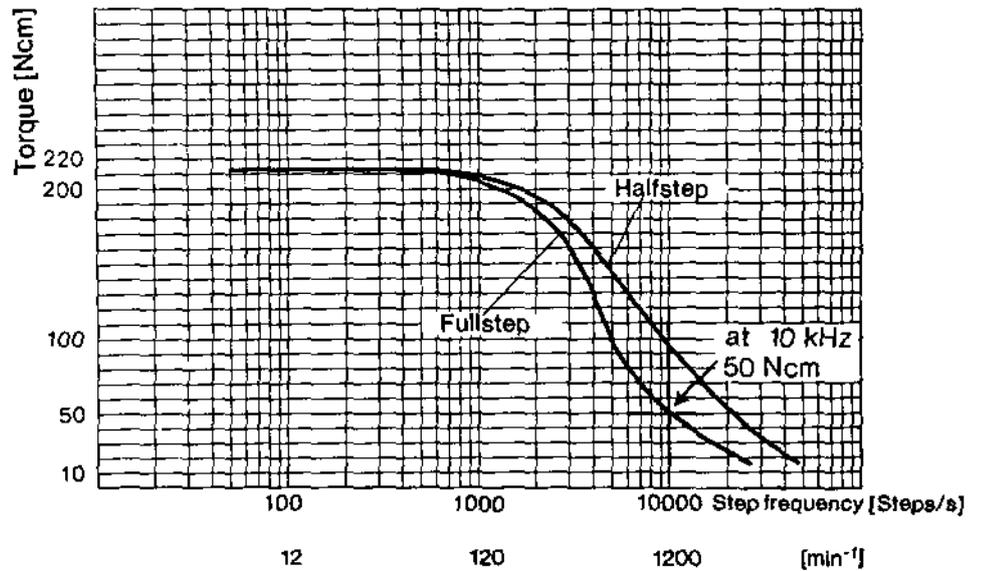
### Data Overview 5-Phase Stepmotor

MOTOR MODEL	▶	RDM 596/50	RDM 599/50	RDM 5913/50
Step angle [Degrees]		0,72	0,72	0,72
Maximum torque [Ncm]		115	210	310
Holding Torque, excited [Ncm]		125	220	400
Max. Power [W] at ... Hz		48 10	60 10	160 8
Moment of inertia of rotor [kgcm <sup>2</sup> ]		0,7	1,2	1,8

### Constant Current Operation (70 V)

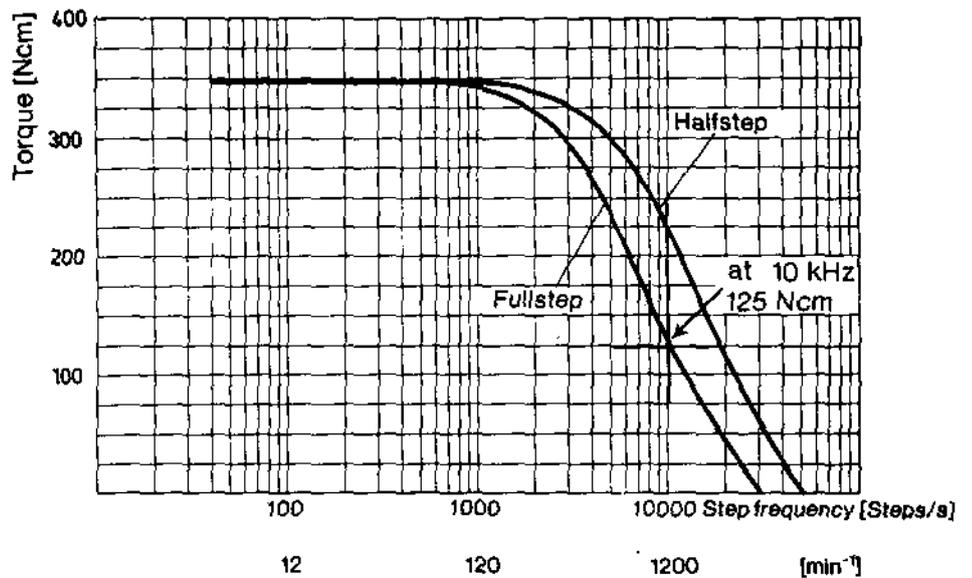
Motor Model **RDM 599/50**

$$R_w = 3.25 \Omega \quad I_w = 1.15 \text{ A}$$



Motor Model **RDM 5913/50**

$$R_w = 1 \Omega \quad I_w = 2.8 \text{ A}$$



For these two motor models:

a) **RDM 599/50**

$$J_{Mot} = 1,2 \text{ kgcm}^2$$

$$M_A = M_{Mot} \text{ (at 10 kHz)} - M_L$$

$$M_A = 50 \text{ Ncm} - 14,4 \text{ Ncm}$$

b) **RDM 5913/50**

$$J_{Mot} = 1,8 \text{ kgcm}^2$$

$$M_A = M_{Mot} \text{ (at 10 kHz)} - M_L$$

$$M_A = 125 \text{ Ncm} - 14,4 \text{ Ncm}$$

## 2.2.7 Calculation of Acceleration Time

For linear acceleration:

**Acceleration Time = Deceleration Time**

$$(11) \quad t_A = J_{\text{tot}} \cdot \frac{2 \cdot \pi \cdot \alpha \cdot f}{360^\circ \cdot M_A \cdot 10^2} \quad [\text{s}]$$

$$(5a) \quad J_{\text{tot}} = J_{\text{ext}} + J_{\text{Mot.}} \quad [\text{kgcm}^2]$$

$$J_{\text{tot}} = 3,043 \text{ kgcm}^2 + 1,2 \text{ kgcm}^2 = 4,243 \text{ kgcm}^2$$

for RDM 599/50

$$J_{\text{tot}} = 3,043 \text{ kgcm}^2 + 1,8 \text{ kgcm}^2 = 4,843 \text{ kgcm}^2$$

for RDM 5913/50

$$(11a) \quad M_A = M_{\text{Mot}} (\text{at } 10 \text{ kHz}) - M_L \quad [\text{Ncm}]$$

$$M_A = 50 \text{ Ncm} - 14,4 \text{ Ncm} = 35,6 \text{ Ncm}$$

for RDM 599/50

$$M_A = 125 \text{ Ncm} - 14,4 \text{ Ncm} = 110,6 \text{ Ncm}$$

for RDM 5913/50

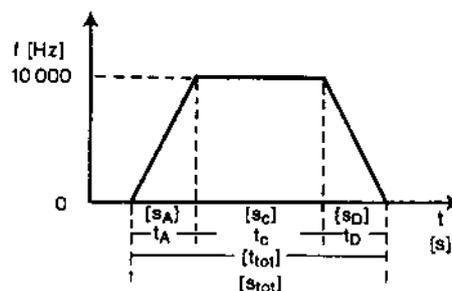
$$t_A = 4,243 \text{ kgcm}^2 \cdot \frac{2 \cdot 3,14 \cdot 0,72^\circ \cdot 10000 \text{ Hz}}{360^\circ \cdot 35,6 \text{ Ncm} \cdot 10^2}$$

$$t_A = 0,148 \text{ s for RDM 599/50}$$

$$t_A = 4,843 \text{ kgcm}^2 \cdot \frac{2 \cdot 3,14 \cdot 0,72^\circ \cdot 10000 \text{ Hz}}{360^\circ \cdot 110,6 \text{ Ncm} \cdot 10^2}$$

$$t_A = 0,0548 \text{ s} \sim \text{for RDM 5913/50}$$

$$t_A \approx 0,055 \text{ s}$$



$t_A$  = Time for acceleration  
 $t_C$  = Time for constant speed  
 $t_D$  = Time for deceleration

## 2.2.8 Distance Traveled

$$(27) \quad s_A = \frac{f \cdot t_A}{2} \quad [\text{Distance in Steps}]$$

$$s_D = \frac{10000 \cdot 0,055}{2} = 275 \text{ Steps for RDM 5913/50}$$

$$\text{During acceleration phase} = 275 \text{ Steps} = s_A$$

$$\text{During deceleration phase} = 275 \text{ Steps} = s_D$$

550 Steps

### 2.2.9 Total Travel Time

Distance of 10 mm  $\hat{=}$  20 · 1 Revolution = 10000 Steps

Spindle pitch h  $\hat{=}$  10 = 10 mm Advance  
 at 1 Revolution  $\hat{=}$  500 Steps  
 i = 20 : 1

$$t_{\text{tot}} = t_A + t_C + t_D \quad [\text{s}]$$

$$t_C = \frac{s_{\text{tot}} - (s_A + s_D)}{f} = \frac{10000 - 550}{10000} = 0,945 \text{ s}$$

$$t_{\text{tot}} = 0,055 + 0,945 + 0,055 = 1,055 \text{ s}$$

Therefore, if a distance of 10 mm is to be traversed in 1 second, the frequency must be increased, in this example to about 11000 Hz.

If the RDM 599/50 had been selected instead of the RDM 5913/50, the total time would be longer and the frequency would have to be increased even further.

### 2.2.10 Frequency Increase to 11 kHz (arbitrarily chosen)

$$(11a) \quad M_A = M_{\text{Mot}} (\text{at } 11 \text{ kHz}) - M_L \quad [\text{Ncm}]$$

$$M_A = 120 \text{ Ncm} - 14,4 \text{ Ncm} = 105,6 \text{ Ncm}$$

$$t_A = 4,843 \text{ kgcm}^2 \frac{2 \cdot 3,14 \cdot 0,72^\circ \cdot 11000 \text{ Hz}}{360^\circ \cdot 105,6 \text{ Ncm} \cdot 10^2}$$

$$t_A = 0,0634 \text{ s}$$

$$(27) \quad s_A = \frac{f \cdot t_A}{2} \quad [\text{Distance in Steps}]$$

$$s_A = \frac{11000 \text{ Hz} \cdot 0,0568 \text{ s}}{2} = 349 \text{ Steps}$$

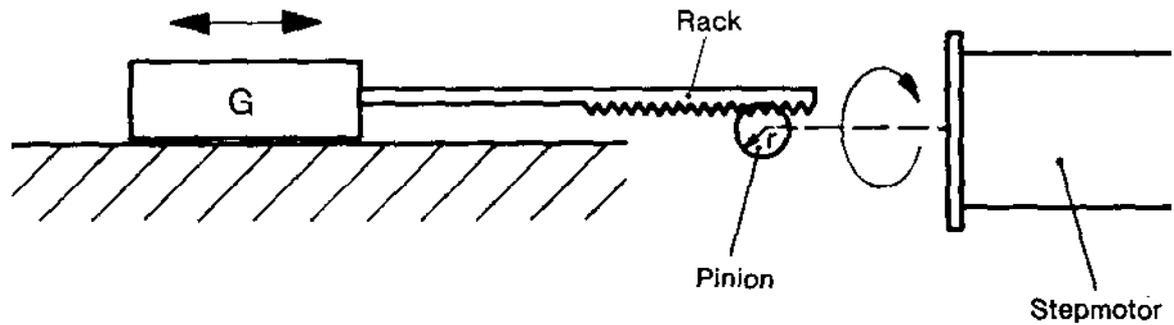
$$s_A + s_D = 698 \text{ Steps}$$

$$t_C = \frac{s_{\text{tot}} - (s_A + s_D)}{f} = \frac{10000 - 698}{11000} = 0,8456 \text{ s}$$

$$t_{\text{tot}} = 0,0634 + 0,8456 + 0,0634 = 0,9724 \text{ s}$$

## 2.3 Calculation: Rack and Pinion Drive

**Example:** Horizontally moved mass, driven by rack and pinion.



### 2.3.1 Known and Required Values

Given:

Weight	$G = 4 \text{ kg}$
Radius	$r = 3 \text{ cm}$
Pinion width	$L = 1,5 \text{ cm}$
Material: Steel	$\gamma = 7,85 \cdot 10^{-3} \text{ kg/cm}^3$
Speed	$v = 400 \text{ cm/s}$
Acceleration Time	$t = 0,5 \text{ s}$

### 2.3.2 Existing Moments of Inertia

$$(19) \quad J_{eq} = G \cdot r^2 \quad [\text{kgcm}^2]$$

$$J_{eq} = 4 \cdot (3)^2 = 36 \text{ kgcm}^2$$

$$J_{Pin} = \frac{1}{2} \cdot \pi \cdot r^4 \cdot L \cdot \gamma = \frac{1}{2} \cdot \pi \cdot 3^4 \cdot 1,5 \cdot 7,85 \cdot 10^{-3}$$

$$J_{Pin} = 1,5 \text{ kgcm}^2$$

Assumed motor model: RDM 5913/50

$$J_{Mot} = 1,8 \text{ kgcm}^2$$

Rotor moment of inertia see catalog data.

$$(5a) \quad J_{tot} = J_{eq} + J_{Pin} + J_{Mot} \quad [\text{kgcm}^2]$$

$$J_{tot} = 36 + 1,5 + 1,8 = 39,3$$

$$J_{tot} \sim 40 \text{ kgcm}^2$$

### 2.3.3 Step Frequency

$$(13a) \quad f = \frac{v \cdot Z \cdot i}{2 \cdot \pi \cdot r} \quad [\text{s}^{-1}]$$

$$f = \frac{400 \text{ cm/s} \cdot 500 \cdot 1}{2 \cdot \pi \cdot 3 \text{ cm}} \approx 10615 \text{ Steps/s}$$

### 2.3.4 Acceleration and Load Torque

$$M_A = J_{\text{tot}} \cdot \frac{f}{t_A} \cdot \frac{2\pi \cdot \alpha}{360^\circ \cdot 10^2}$$

$$M_A = 40 \cdot \frac{10610}{0,5} \cdot \frac{2 \cdot 3,14 \cdot 0,72}{360^\circ \cdot 10^2} = 107 \text{ Ncm}$$

$$M_A = 107 \text{ Ncm}$$

$$(21) \quad M_{\text{tot}} = M_A + M_L \quad [\text{Ncm}]$$

$$M_{\text{tot}} = 107 + 0,6 = 107,6 \sim 108 \text{ Ncm}$$

$$(22) \quad M_L \sim G \cdot \mu \cdot r \quad [\text{Ncm}]$$

$$M_L \sim 40 \cdot 0,005 \cdot 3 = 0,6 \text{ Ncm}$$

### 2.3.5 Verification of Data

Model RDM 5913/50 is too weak, it would operate at its very performance limit (100%).

$$M = 120 \text{ Ncm at } 10.6 \text{ kHz}$$

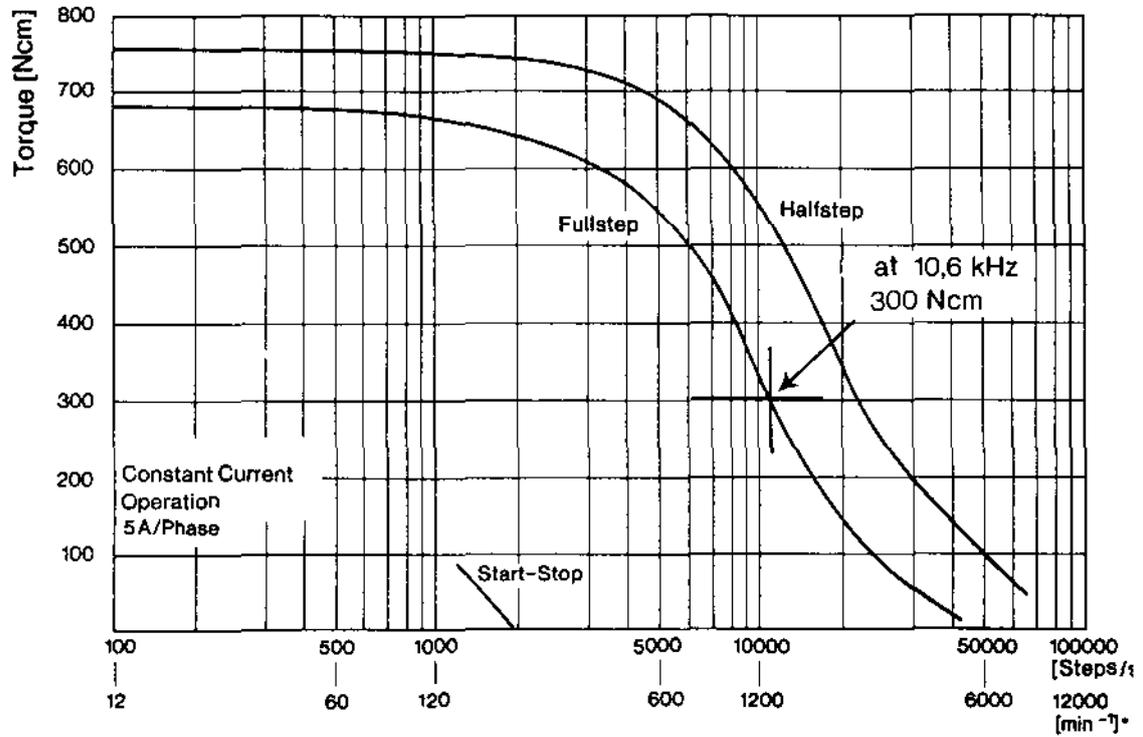
This can be remedied in the following ways:

- a) Increase acceleration time by about 20%, this would reduce M to about 95 Ncm  
or
- b) Reduce speed  $v = 400 \text{ cm/s}$  by about 20%, this would reduce M to about 92 Ncm  
or
- c) Use next larger motor model (RDM 51117/50). This model would provide a torque  $M_{\text{Mot}}$  of about 300 Ncm at 10.6 kHz.

### 2.3.6 Torque Curves

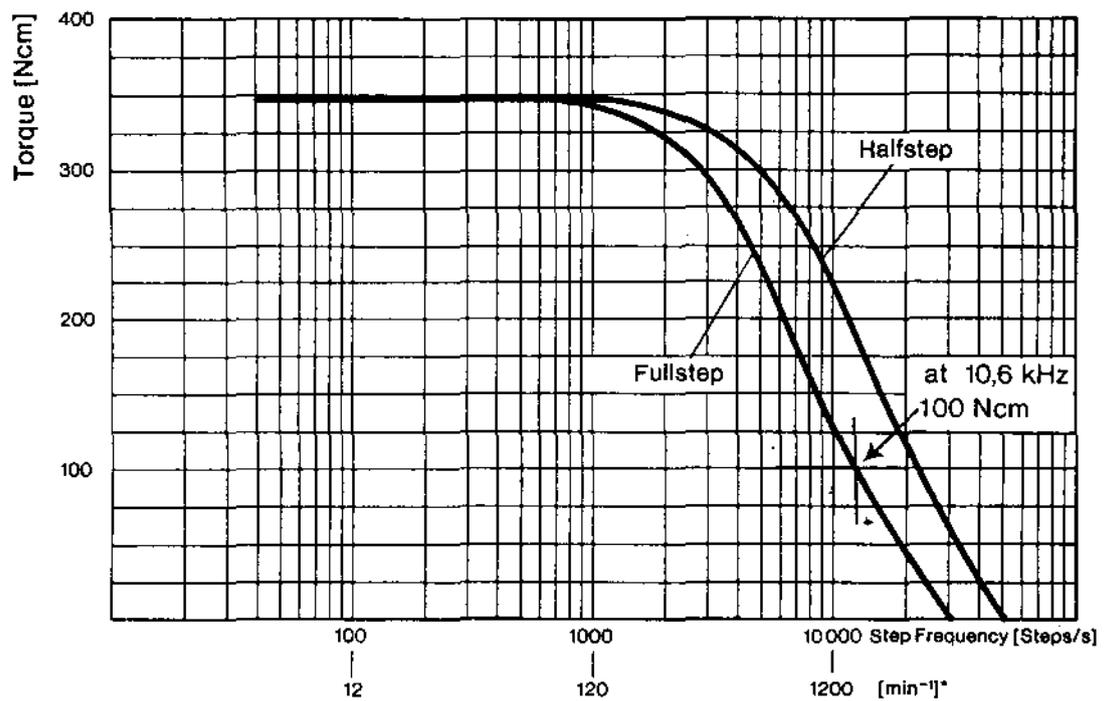
Motor Model **RDM 51117/50**

$R_w = 0.3 \Omega$     $I_w = 5 \text{ A}$



Motor Model **RDM 5913/50**

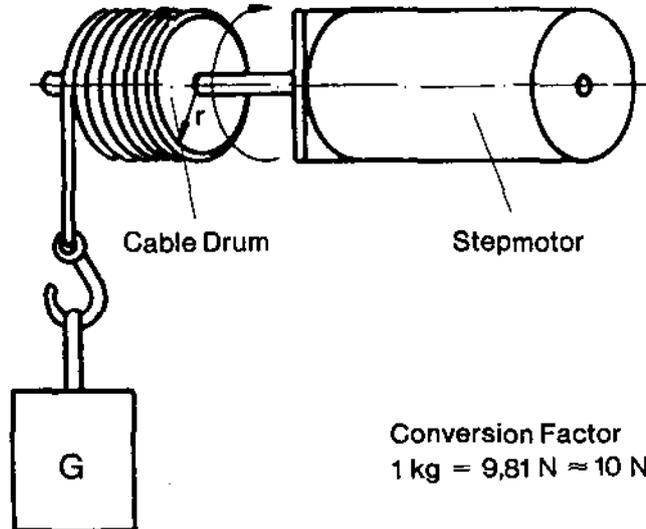
$R_w = 1 \Omega$     $I_w = 2.8 \text{ A}$



\* For fullstep 0.72°

## 2.4 Calculation: Drum Drive

### Lifting a Weight by Cable and Drum



#### 2.4.1 Known and Required Values

Weight	$G = 4 \text{ kg}$
Drum radius	$r = 3 \text{ cm}$
Lifting speed	$v = 400 \text{ cm/s}$
Acceleration time	$t_A = 0,5 \text{ s}$
Drum moment of inertia	$J_{Dr} = 10 \text{ kgcm}^2$

#### 2.4.2 Existing Moments of Inertia

$$J_{eq} = G \cdot r^2 \text{ [kgcm}^2\text{]}$$

$$J_{eq} = 4 \text{ kg} \cdot (3 \text{ cm})^2 = 36 \text{ kgcm}^2$$

$$J_{Dr} = 10 \text{ kgcm}^2$$

$$J_{ext} = 46 \text{ kgcm}^2$$

#### 2.4.3 Existing Load Torque

$$(23) \quad M_L = G \cdot r \text{ [Ncm]}$$

$$M_L = 4 \text{ kg} \cdot 3 \text{ cm} = 12 \text{ kgcm} \triangleq 120 \text{ Ncm}$$

#### 2.4.4 Operating Frequency

$$(13) \quad v = \frac{h}{Z \cdot i} \cdot f \text{ [cm/s]}$$

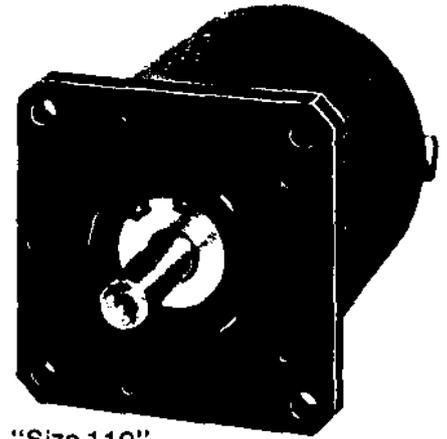
$$(13a) \quad f = \frac{v \cdot Z \cdot i}{2 \cdot \pi \cdot r} \text{ [s}^{-1}\text{]}$$

$$f = \frac{400 \text{ cm/s} \cdot 500 \text{ Steps} \cdot 1}{2 \cdot 3,14 \cdot 3 \text{ cm}} = 10615 \text{ Steps/s} \quad \triangle [s^{-1}]$$

#### 2.4.5 Values Determining the Motor Size

- 1)  $M_L = 120 \text{ Ncm}$
- 2)  $J_{ext} = 46 \text{ kgcm}^2$
- 3)  $f = 10615 \text{ s}^{-1}$

## 2.4.6 Determination of Motor Size



"Size 110"

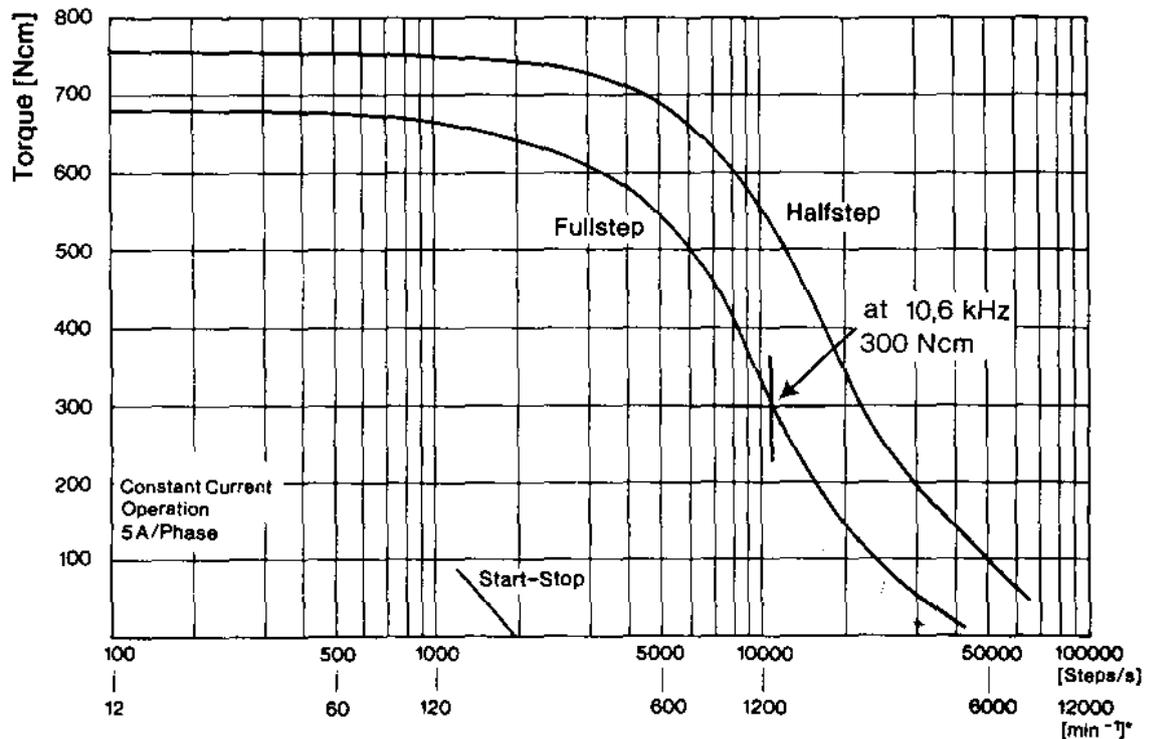
### 5-Phase Stepmotor Data Overview

The motor data are contained in our Catalog No. 250/... "5-Phase Stepmotor System" in the following arrangement

MOTOR MODEL	RDM 51117/50	RDM 51122/50
Step angle [Degrees]	0,72	0,72
Maximum torque [Ncm]	700	1000
Holding Torque, excited [Ncm]	750	1100
Max. Power [W] at ... Hz	430 15	465 5
Moment of inertia of rotor [kgcm <sup>2</sup> ]	7,5	11,5

Motor Model: **RDM 51117/50** Constant Current Operation (90 V)

$$R_w = 0,3 \Omega \quad I_w = 5 \text{ A}$$



\* For fullstep 0.72°

Based on this data, the RDM 51117/50 was selected  $M_{Mot}$  at 10,6 kHz 300 Ncm  
 $J_{Mot}$  7,5 kgcm<sup>2</sup>

#### 2.4.7 Acceleration Torque

$$(18) \quad M_A = J_{tot} \cdot \frac{f \cdot 2 \cdot \pi \cdot a}{t_A \cdot 360^\circ \cdot 10^2} \quad [\text{Ncm}]$$

$$(5a) \quad J_{tot} = J_{ext} + J_{Mot} \quad [\text{kgcm}^2]$$

$$J_{tot} = 46 \text{ kgcm}^2 + 7,5 \text{ kgcm}^2 = 53,5 \text{ kgcm}^2$$

$$M_A = 53,5 \cdot \frac{10615 \cdot 2 \cdot \pi \cdot 0,72}{0,5 \cdot 360 \cdot 10^2} = 142,66 \text{ Ncm}$$

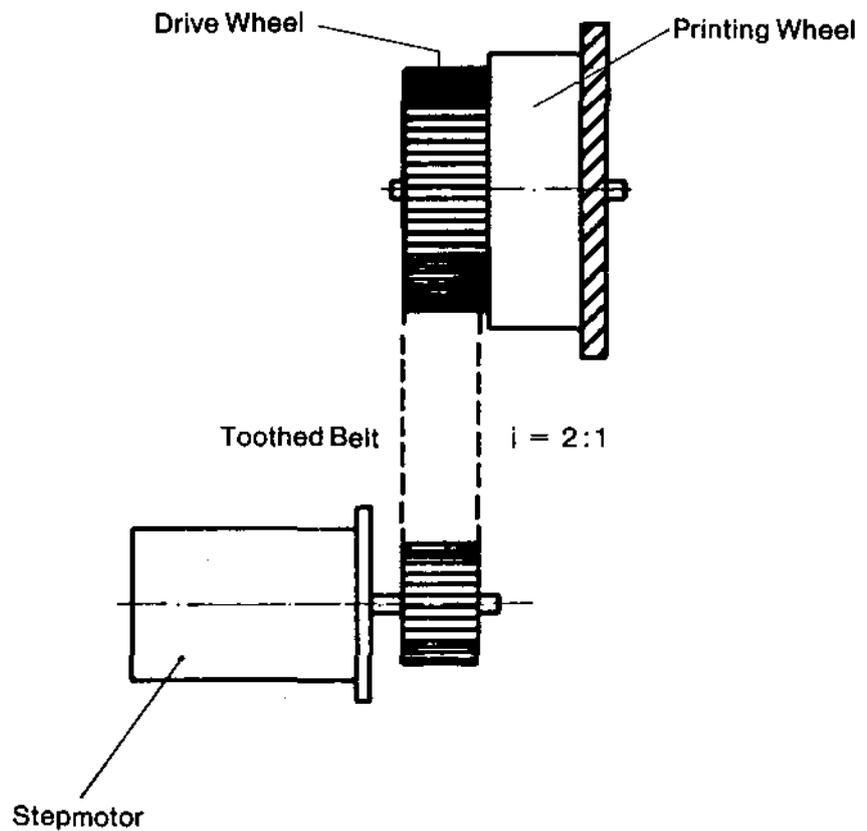
#### 2.4.8 Total Torque to be Gerated

$$(21) \quad M_{tot} = M_L + M_A \quad [\text{Ncm}]$$

$$M_{tot} = 120 \text{ Ncm} + 142,66 \text{ Ncm} = 262,66 \text{ Ncm}$$

*i.e. Model RDM 51 117/50 with 300 Ncm is sufficient, it provides a reserve of 37.3 Ncm = 13%*

## 2.5 Calculation: Flywheel Drive



### 2.5.1 Data

Printing Wheel		Drive Wheel		Motor Pinion	
Diameter	140 mm	Diameter	90 mm	Diameter	45 mm
Width (L)	90 mm	Width (L)	25 mm	Width (L)	25 mm
Material	Steel	Material	Aluminum	Material	Aluminum
		Number of Teeth	24	Number of Teeth	12

Reduction ratio 2 : 1 by toothed belt.

Printing wheel setting time 0.3 s for 180°.

## 2.5.2 Moments of Inertia

$$(6) \quad J_{\text{rot}} = \frac{1}{2} \pi \cdot r^4 \cdot L \cdot \gamma \quad [\text{kgcm}^2]$$

For steel

$$(7) \quad J_{\text{rot}} = 7,72 \cdot 10^{-4} \cdot d^4 \cdot L \quad [\text{kgcm}^2]$$

For Aluminum:

$$(7a) \quad J_{\text{rot}} = 2,7 \cdot 10^{-4} \cdot d^4 \cdot L \quad [\text{kgcm}^2]$$

If a transmission is used, Formulas 6 - 7 a must be multiplied by  $\frac{1}{i^2}$ .

The formulas then become:

$$J_{\text{ext}} = (J_{\text{Print}} + J_{\text{Drive}}) \cdot \frac{1}{i^2} + J_{\text{Pinion}}$$

$$J_{\text{Print Wheel}} = 7,72 \cdot 10^{-4} \cdot (14 \text{ cm})^4 \cdot 9 = 267 \text{ kgcm}^2$$

$$J_{\text{Drive Wheel}} = 2,7 \cdot 10^{-4} \cdot 9^4 \cdot 2,5 = 4,43 \text{ kgcm}^2$$

$$J_{\text{Pinion}} = 2,7 \cdot 10^{-4} \cdot 4,5^4 \cdot 2,5 = 0,28 \text{ kgcm}^2$$

$$J_{\text{ext}} = (267 + 4,43) \cdot \frac{1}{2^2} + 0,28 = 68,13 \approx 70 \text{ kgcm}^2$$

$$(5a) \quad J_{\text{tot}} = J_{\text{ext}} + J_{\text{Mot}} \quad [\text{kgcm}^2]$$

## 2.5.3 Acceleration Torque

$$(18) \quad M_A = J_{\text{tot}} \cdot \frac{f \cdot 2 \cdot \pi \cdot \alpha}{t_A \cdot 360^\circ \cdot 10^2} \quad [\text{Ncm}]$$

## 2.5.4 Acceleration Time

$$(11) \quad t_A = J_{\text{tot}} \cdot \frac{2 \pi \cdot \alpha \cdot f}{360^\circ \cdot M_A \cdot 10^2} \quad [\text{s}]$$

$f$  is calculated from the requirement  $i = 2 : 1$  and  $t = 0.3 \text{ s}$  for a  $180^\circ$  rotation of the printing wheel, i. e. the motor must execute a rotation of  $360^\circ$  in  $0.3 \text{ s}$ . For a 5-phase motor operating in the fullstep mode, this means 500 steps.

$$\text{The frequency is then } \frac{500 \text{ Steps}}{0,3 \text{ s}} = 1667 \text{ Hz}$$

However, as the motor must accelerate and decelerate, a higher frequency must be chosen. We assume here  $f = 2500 \text{ Hz}$ .

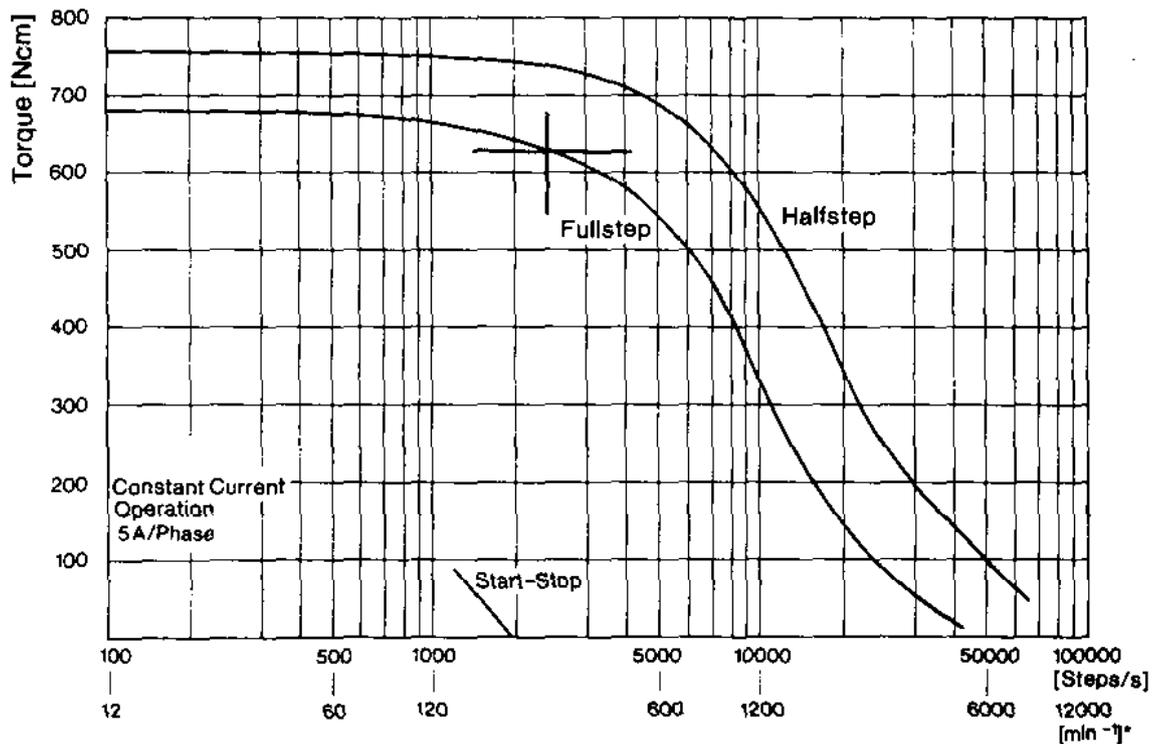
The RDM 51117/50 is selected because it is the only motor capable of accelerating the 70 kgcm<sup>2</sup>

$$J_{\text{tot}} = J_{\text{ext}} + J_{\text{Mot}} = 70 \text{ kgcm}^2 + 7,5 \text{ kgcm}^2 = 77,5 \text{ kgcm}^2$$

$M_{\text{Mot}}$  = the available torque at 2500 Hz according curve approx. 600 Ncm

Motor Model: **RDM 51117/50**

$R_w = 0,3 \Omega$   $I_w = 5 \text{ A}$



\* For fullstep 0.72°

$$t_A = 77,5 \text{ kgcm}^2 \cdot \frac{2 \cdot 3,14 \cdot 0,72^\circ \cdot 2500 \text{ s}^{-1}}{360^\circ \cdot 600 \text{ Ncm} \cdot 10^2} = \underline{\underline{0,04 \text{ s}}}$$

$$(27) \quad s_A = \frac{f \cdot t_A}{2} \text{ [Steps]}$$

$$s_A = \frac{2500 \cdot 0,04}{2} = 50 \text{ Steps}$$

### 2.5.5 Total Time

As acceleration = deceleration,  $s_A = s_D = 2 \times 50 = 100 \text{ Steps}$

Total distance  $s_{\text{tot}} = 180^\circ$  at  $i = 2 : 1$ , 500 Steps

$$t_{\text{tot}} = t_A + t_c + t_D \text{ [s]}$$

$$t_c = \frac{s_{\text{tot}} - (s_A + s_D)}{f} = \frac{500 - 100}{2500} = 0,16 \text{ s}$$

$$t_{\text{tot}} = 0,04 + 0,16 + 0,04 = 0,24 \text{ s}$$